Society of Engineering Science 51st Annual Technical Meeting

1–3 October 2014

Purdue University, West Lafayette, Indiana, USA

Nonlinear deflection of a fixed–fixed hyperelastic beam under extreme stretch

Wissman, James, jwissman@andrew.cmu.edu; Majidi, Carmel, Carnegie Mellon University, United States

ABSTRACT

We present a model that describes the deflection of a prestretched elastic beam under planar loading conditions. The kinematics is represented with a Cosserat directed curve with an extensible arc length. The strain within the cross-section of the rod is composed of bending strain and stretch of the centerline. Given that bending strains are relatively small, an effective flexural rigidity is defined based on the spatial cross-section and the slope of the hyperelastic stress–strain curve evaluated for the stretch along the neutral axis. Initially, no additional strain is assumed as the beam is deflected, and small deflections allow for the application of the small angle approximation. The solution is reduced to a fourth-order ODE that resembles an Euler–Bernoulli beam equation with a correction term accounting for axial loading. The model is further examined for the case of a stretched fixed–fixed beam under a point load applied at the center. An iterative approach is taken to accommodate further stretching as the load is applied. Experimental results are then compared to the theory. Silicone rubber beams are fixed to rigid blocks capable of shifting longitudinally to induce desired prestretches. Under various stretched conditions, the beams are then deflected vertically with a wedge while recording data on position and force. Although the Neo-Hookean constitutive model overestimates deflection at higher prestrains, a four-parameter Ogden model captures the behavior well and is in good agreement with experimental measurements for prestrains of up to 200%. The results of this analysis have applications in the area of soft robotics and electronics, where devices such as elastic microelectromechanical switches will be expected to function regardless of stretch.